The Effectiveness of Visualization of Proofs in Learning Mathematics by Using Discovery Learning Viewed from Conceptual Understanding

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Abstract

The purpose of this study is to explore the effectiveness of the use of visualization of proofs upon discovery learning models in mathematics learning in terms of understanding concepts. This study is an experimental design which used quantitative methods to obtain data on student conceptual understanding. The sampling technique used was stratified random sampling. The population sample used in this study was 11th grade secondary students, chosen from 11 IPA 2 of SMAN 8 Yogyakarta, 11 IPA 3 of SMAN 8 Yogyakarta, 11 IPA 3 of SMAN 2 Yogyakarta, 11 IPA 4 of SMAN 2 Yogyakarta, 11 IPA 1 of SMAN 11 Yogyakarta, and 11 IPA 2 of SMAN 11 Yogyakarta. In each school, two classes were chosen with one class was given a discovery learning treatment with visualization of proofs (PWW) and the other class was given a discovery learning treatment without visualization of proofs. The measurement instrument used in this study was an essay test instrument with five questions. Discovery learning is said to be effective if the average value of conceptual understanding is at least 75. Based on the results of this study, although the data obtained did not meet the assumptions of normality, the number of members of the sample were more than 30 so that the data analysis could use parametric statistics. Using a hypothesis testing with a significance level of 0.05, it was found that the use of visualization of proofs was effective in mathematics discovery learning models if it was viewed from conceptual understanding.

Keywords: conceptual understanding, visualization of proofs, discovery learning.

Introduction

Various studies reveal that Indonesian classroom learning process in general is not conducted interactively so that it fails to foster students' creativity, critical thinking, and analytical skills (Kemdikbud, 2015). Thus, competence and ownership of learning by students as a result of the process are not encouraged. Student learning outcomes are also not encouraging, with the National Examination (UN) in 2016 and 2017 showed that the level of mathematics mastery of high school students in Natural Sciences in Yogyakarta was still below 65% (PUSPENDIK, 2016, 2017).

The Minister of Education and Culture (MoEC) Regulation No. 22 of 2016 on basic and secondary education process standards states that:

> … the process of learning mathematics in educational units should be interactive, inspirational, fun, challenging, motivate students to participate actively, and provide sufficient space for initiative, creativity, and independence according to students talents, interests, and physical and psychological development (Kementerian Pendidikan dan Kebudayaan, 2016, p. 6).

Therefore, each education provider is required to make learning plans that employ the learning process standards with the aim at increasing student competencies. The provision of understanding knowledge is expected to occur through the activities of knowing, understanding, applying, analysing, evaluating, and creating, while the skills components are trained through the activities of observing, asking, trying, reasoning, serving, and creating.

One learning model that fits the demands of the MoEC Regulation is discovery learning. This learning model emphasizes students' discovery of previously unknown concepts or principles engineered by teacher. In discovery learning, the material to be presented is not delivered in the final form but students are encouraged to identify what they want to know, find their own information, organize, form or construct what they know and understand in a final form under the guidance of the teacher (PUSPENDIK, 2014).

The use of discovery learning aims to: (1) change learning conditions from passive to active and creative; (2) change teacher-oriented teaching to student-oriented learning; and (3) change the expository style where students only receive overall information from the teacher to the discovery style where students find their own information. Discovery learning has proven advantages compared to traditional or expository learning styles. Students who learn by exploration and discovery will benefit cognitively and learning becomes meaningful (Rieber, Tzeng, & Tribble, 2004). Every concept they discover, struggle with, and apply will become a part of their own deep understanding. Discovery learning has demonstrated an increase in student successful learning compared to traditional learning methods (Balım, 2009; Prince & Felder, 2006)

Discovery learning shares the same principles as inquiry learning and problem-solving, so that it can also be called an inquiry-discovery learning model (Syah, 2014). At a philosophical level there are differences, but at a teacher level they merge. To apply the discovery learning method in classrooms, a teacher must facilitate activities that include the following steps: stimulation (simulation), problem statement (statement/identification of problem, consider the variables), data collection (what data needs to be collected), data processing, verification, and generalization (drawing conclusions/generalizations) (Aziz, Tarmedi, & Kusmarni, 2018; PUSPENDIK, 2014; Syah, 2014).

According to NCTM, analytics (reasoning and proof) are among the standard processes in learning mathematics (NCTM, 2000). Thus, in the process of learning mathematics, students should be trained to be able to prepare arguments and prove a mathematical statement. In compiling mathematical arguments or evidence, students must be able to identify the components contained in the mathematical statement and its properties. Teacher has an important role to help students to be able to understand how mathematical arguments and evidence are arranged (Hanna & Villiers, 2008; Knipping, 2004). Learning mathematics with mathematical argumentation and proof is very important. Through mathematical argumentation activities and proving mathematical statements students practise: (1) ensuring the truth of a mathematical statement or formula; (2) gaining an in-depth understanding of a mathematical concept; (3) conveying or communicating ideas; (4) thinking critically and deeply; (5) building a mathematical and systematic framework (Hanna, 2014). Besides proving activities, mathematical proof itself can be a very useful media in learning mathematics. When students are given a mathematical proof and asked to analyse and explain it, the process trains them to think analytically and to communicate their ideas or understanding of the evidence.

Mathematics is an abstract object that only exists in mind, and this makes mathematical proofs difficult for students to understand. Learning using mathematical proofs can be carried out interactively, inspirationally, challenging, with fun and motivation to encourage students to participate actively. This can be achieved by using concrete forms of objects or abstract mathematical proofs. Visualization of an object by a student helps in the learning and understanding processes (Philips, Norris, & Macnab, 2010). The object of visualization can be in the form of pictures, schematic diagrams, computer simulations, or videos. In mathematics learning activities, an image or visual object often helps students in understanding mathematical concepts by providing important clues to solving problems. An image can also function as a concrete picture of an abstract mathematical concept (Aso, 2001).

Understanding concepts in mathematics also involves understanding operations and mathematical relations. Understanding concepts is an integrated and functional understanding of mathematical ideas. Students with conceptual understanding understand more about implied facts and methods. They understand why mathematical ideas are important and which types of context are useful. Students have organized their knowledge into a comprehensive whole, which gives them the opportunity to learn new ideas by connecting them with what they already know (National Research Council, 2010). Understanding the concepts in mathematics involves a thorough understanding of the basic and fundamental concepts behind the algorithm being performed. Thus, this includes a situation where students can rewrite a formula and proof without memorizing the process (Hasnida, Ghazali, & Zakaria, 2011).

Students' understanding of concepts forms the basis for learning at a higher level. Concepts can be divided into two types, namely concrete concepts and defined concepts (Nitko & Brookhart, 2011). The concrete concept refers to a classification in which each member of the class is concrete, that is, it can be physically captured by some of the five senses (can be seen, heard, touched, tasted, and smelled). Whereas defined concepts refer to classifications, whose members can be defined in the same way by traits that are not real and often involve relationships with other concepts. Defined concepts are often called abstract concepts or relational concepts. Someone understands a concept if s/he can (1) mention the definition or understanding of the concept; (2) give an example of the concept; (3) distinguish between examples and non-examples based on the concept; and (4) identify components and non-components and mention the relationship of the components contained in the concept. However, by being able to do all four things, one cannot be said to comprehend the concept in depth. A student understands concepts in depth if s/he: (1) is able to use concepts to solve problems; (2) associates a concept with other concepts or principles and generalize what has been learned; and (3) uses concepts to learn new material (Nitko & Brookhart, 2011).

To summarise, learning mathematics through discovery learning can facilitate students to find the relationship of patterns and the nature of observations until they can conclude and construct their own understanding of a concept. Learning mathematics with visualization of proofs can stimulate students visually to think mathematically. This will assist students in linking previously known mathematical concepts to find the truth of a concept being studied. The similarity of these principles is that both facilitate students to find and link concepts that have been studied before to learn new concepts. This study will use the visualization of proofs combined with the discovery learning model to stimulate students to learn. It will investigate the effectiveness of both in mathematics learning in terms of conceptual understanding.

Methods

This research is an experimental research (Berger, Maurer, & Celli, 2018, p. 18) which has the following characteristics: (1) manipulation by researchers of one or more independent variables; (2) the use of controls such as random sampling of research subjects to minimize the effects of disturbance variables; and (3) careful observation or measurement of one or more dependent variables.

The population of this study was 11th grade students in Yogyakarta and stratified random sampling (Walpole, Myers, & Ye, 2007) was used. There are eleven public high schools (SMAN) in Yogyakarta. Based on the results of the 2017 National Examination, four high schools ranked at the bottom were SMAN 4, SMAN 6, SMAN 10, and SMAN 11. Four high schools in the middle rank were SMAN 2, SMAN 5, SMAN 7 and SMAN 9. Three high schools located at the top were SMAN 1, SMAN 3, and SMAN 8. The samples in this study were from a school that was chosen randomly, and from each school two classes of 11th grade were chosen randomly. The selected sample in this study were 11 IPA 2 and 11 IPA 3 of SMAN 8 Yogyakarta, 11 IPA 3 and 11 IPA 4 of SMAN 2 Yogyakarta, and 11 IPA 1 and 11 IPA 2 of SMAN 11 Yogyakarta.

Reliability estimates used are internal-consistency estimates of reliability using the reliability coefficient α as the estimated value of reliability (Allen & Yen, 1979). The test instrument used in this study was an essay test, the test instrument was said to be reliable if the estimated value of the reliability was more than 0.65 (Nitko & Brookhart, 2011). A trial to find the reliability estimation of the instrument was conducted for 30 students of 11 IPA 1 of SMAN 8 Yogyakarta. Based on the results of this trial it appeared that the Cronbach α coefficient value was 0.888 so it can be said that the test instruments used in this study are reliable.

Results and Discussion

The variable measured in this study is conceptual understanding. The variable was measured using an essay test of five questions. Measurement of this variable is carried out twice, namely a pre-test and a post-test. The results of measurement are presented in the following table.

Table 1

Summary of Test Results on Conceptual Understanding

No	School	Group	Stats	Conceptual Understanding		
				Pre-test	Post-test	Differences
1	All	Experiment	Means	23.02	92.28	69.27
			Std Dev	20.49	11.28	22.99
		Control	Means	27.17	69.39	42.23
			Std Dev	18.36	24.37	30.20
2	SMAN 8	Experiment	Means	14.71	87.15	72.45
			Std Dev	17.21	16.32	22.37
		Control	Means	23.06	88.39	65.33
			Std Dev	19.64	17.46	27.02
3	SMAN 2	Experiment	Means	29.68	93.42	63.74

No	School	Group	Stats	Conceptual Understanding		
				Pre-test	Post-test	Differences
			Std Dev	17.78	8.19	17.52
		Control	Means	26.49	52.97	26.48
			Std Dev	17.68	21.53	27.92
4	SMAN 11	Experiment	Means	23.90	95.07	71.16
			Std Dev	23.08	7.58	26.89
		Control	Means	32.51	80.34	47.83
			Std Dev	18.11	13	19.52

Based on Table 1, there is a significant increase in the assessment of conceptual understanding from the pre-test to the post-test. In the experimental class the value of conceptual understanding increased by 69.27 points from 23.02 to 92.28, while in the control class the value of conceptual understanding increased by 42.23 points from 27.17 to 69.39 points.

To find out whether the data obtained meets the assumptions that the sample data comes from normally distributed populations, the data was tested using the Kolmogorov-Smirnov Test with the help of SPSS software.

Table 2
Recapitulation of Test Results for the Normality of Experiment Group Data

No	School	Variable	Asym .Sig α	Normality Assumption
		Pre-conceptual Understanding	0,067	fulfilled
1	All	Post-conceptual Understanding	0,000	unfulfilled
		Difference	0,253	fulfilled
		Pre-conceptual Understanding	0,410	fulfilled
2	SMAN 8	Post-conceptual Understanding	0,176	fulfilled
		Difference	0,318	fulfilled
		Pre-conceptual Understanding	0,431	fulfilled
3	SMAN 2	Post-conceptual Understanding	0,085	fulfilled
		Difference	0,664	fulfilled
		Pre-conceptual Understanding	0,073	fulfilled
4	SMAN 11	Post-conceptual Understanding	0,003	unfulfilled
		Difference	0,429	fulfilled

Based on the normality assumption test where an average value of at least 75 for conceptual understanding is used, what met the normality assumption was the experimental groups in SMAN 8 and SMAN 2. The parametric statistics used were the t test. The t test is done by taking the initial hypothesis that the average value obtained reaches the specified minimum value, and the alternative hypothesis is the average value obtained is less than the specified minimum value. The decision criteria in this test is that if t greater than -1,672 then the initial hypothesis can be accepted. The following table is the result of SPSS data processing.

Table 4
Recapitulation of One Sample T-test of Post-test of Overall Experimental Class, SMAN 8 and SMAN 2

No	School	H_0	t	$t_{0,05;58}$	Decision
1	All	$\mu \geq 75$	11,769	-1,672	H_0 Accepted
2	SMAN 8	$\mu \geq 75$	3,069	-1,672	H_0 Accepted
3	SMAN 2	$\mu \geq 75$	9,547	-1,672	H_0 Accepted

Based on the normality assumption test, it was found that the experimental class data in SMAN 11 did not meet the normality assumption, so to test whether the average value of at least 75 for conceptual understanding, nonparametric statistics was used. The nonparametric statistics used was the sign tests. The sign test was done by taking the initial hypothesis where the average value obtained reaches the specified minimum value ($H_0 : \mu \geq 75$), and the alternative hypothesis is where the average value obtained is less than the specified minimum value ($H_1 : \mu < 75$). Based on the data samples in this class of 24 (n = 24), if $2\sum b\left(x, n, \frac{1}{2}\right) \geq 0.05$ then $H_0 : \mu \geq 75$ is accepted, otherwise $H_0 : \mu \geq 75$ is rejected. Values of $2\sum b\left(x, n, \frac{1}{2}\right)$ are obtained using the help of Microsoft Excel software by using the function "= BINOM.DIST (x; n; 0.5; TRUE)". The result is $2\sum b\left(x, n, \frac{1}{2}\right) = 0.999 > 0.05$ so $H_0 : \mu \geq 75$ is accepted.

Based on this hypothesis test, it can be concluded that the average value of conceptual understanding in both the high school overall and the high school in the top rank ranges, in the middle rank ranges, and the bottom rank ranges of at least 75. The results indicate that the use of visualization of proofs on discovery learning models in mathematics learning is effective in terms of conceptual understanding.

Learning mathematics by using visualization of proofs, students are required to investigate the truth of the images of visualization of proofs. Students are required to identify the concepts contained in the visualization of proofs to be able to know if they are true. Students are then required to look for and find connections between these concepts so that they find the truth of the theorems or formulas listed in the visualizations. This requires the ability to comprehend the concepts deeply. Deep understanding of concepts includes using concepts to solve problems, finding relationships between concepts, and using concepts to learn new material (Nitko & Brookhart, 2011).

These results indicate that a mathematical proof plays an important role in learning mathematics and understanding mathematics (Hanna, 2000; Knuth, 2002). Mathematical proofs presented in the form of images or visualizations can also play an important role in learning mathematics (Alsina & Nelsen, 2010). When students observe mathematical proofs presented in the form of visualizations, students are trained to analyse, arrange arguments, ultimately find and understand the mathematical concepts contained within, and re-communicate the mathematical concepts that have been learned.

Conclusions

Based on the results and discussion of study, it can be concluded that the use of visualization of proofs in discovery learning models in the process of learning mathematics is effective to help students to understand mathematical concepts. This is because visualization of proofs presents related concepts and the relationship between these concepts in the form of images or visuals. These images and visuals help students to find concepts and the relationship between them. Due to the sample size, further research is needed before these findings can be generalised to a larger population.

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